PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

A Theory of the Lifted Temperature Minimum on Calm Clear Nights

A. S. Vasudeva Murthy, J. Srinivasan and R. Narasimha

Phil. Trans. R. Soc. Lond. A 1993 **344**, 183-206
doi: 10.1098/rsta.1993.0087

A theory of the lifted temperature minimum on calm clear nights

By A. S. Vasudeva Murthy[1], J. Srinivasan[2] and R. Narasimha[3]

[1] *TIFR Centre, Indian Institute of Science, Bangalore 560 012, India*

[2] *Centre for Atmospheric Sciences, Indian Institute of Science, Bangalore 560 012, India*

[3] *Centre for Atmospheric Sciences, Indian Institute of Science, Bangalore 560 012, India, and National Aeronautical Laboratory, Kodihalli, Bangalore 560 017, India*

Numerous reports from several parts of the world have confirmed that on calm clear nights a minimum in air temperature can occur just above ground, at heights of the order of $\frac{1}{2}$ m or less. This phenomenon, first observed by Ramdas & Atmanathan (1932), carries the associated paradox of an apparently unstable layer that sustains itself for several hours, and has not so far been satisfactorily explained. We formulate here a theory that considers energy balance between radiation, conduction and free or forced convection in humid air, with surface temperature, humidity and wind incorporated into an appropriate mathematical model as parameters. A complete numerical solution of the coupled air–soil problem is used to validate an approach that specifies the surface temperature boundary condition through a cooling rate parameter. Utilizing a flux-emissivity scheme for computing radiative transfer, the model is numerically solved for various values of turbulent friction velocity. It is shown that a lifted minimum is predicted by the model for values of ground emissivity not too close to unity, and for sufficiently low surface cooling rates and eddy transport. Agreement with observation for reasonable values of the parameters is demonstrated. A heuristic argument is offered to show that radiation substantially increases the critical Rayleigh number for convection, thus circumventing or weakening Rayleigh–Bénard instability. The model highlights the key role played by two parameters generally ignored in explanations of the phenomenon, namely surface emissivity and soil thermal conductivity, and shows that it is unnecessary to invoke the presence of such particulate constituents as haze to produce a lifted minimum.

1. Introduction

Nearly 60 years ago Ramdas & Atmanathan (1932) reported temperature measurements at Poona and three other Indian stations showing that, on calm clear nights, air may be cooler than ground by a few degrees at heights ranging up to about 1 m (figure 1). The report caused considerable surprise for several reasons. First of all it went counter to the prevailing view that following sunset a temperature inversion always develops at ground. This view has apparent support from normal observations of air temperature (see Sutton 1953, p. 190), but it must be remembered that such observations usually stop at the standard screen height of 4 ft (≈ 1.22 m). Even below this height, however, while rapid ground cooling after sunset may be expected

Phil. Trans. R. Soc. Lond. A (1993) **344**, 183–206

Printed in Great Britain

9 Vol. 344. A (16 August 1993)

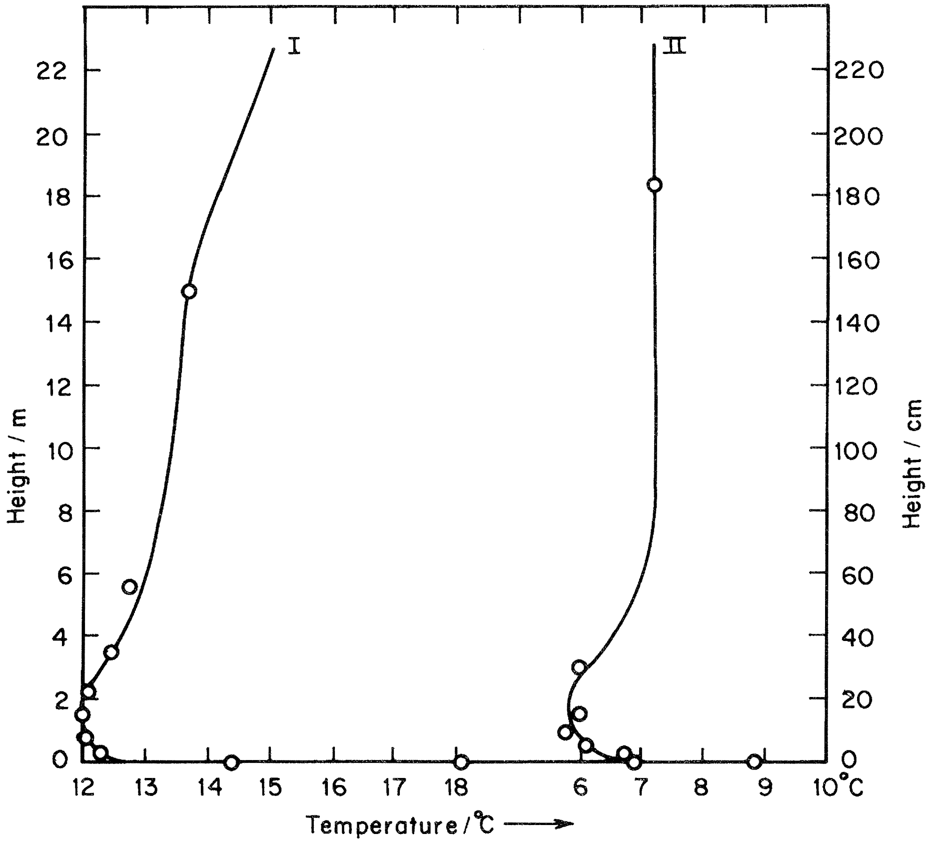


Figure 1. Temperature profiles with lifted minimum, as reported by Ramdas & Atmanathan (1932). I, Poona, 29 Nov 1931, 0600 h; II, Agra, 29 Nov 1931, 0503 h.

(by diffusion and mixing) to cause similar cooling in the air layers closest to ground, there is no obvious reason why air temperature should fall *below* that at ground. Secondly, a temperature minimum above ground should lead to Rayleigh–Bénard instability, so it is not clear how it can be sustained for several hours (virtually till sunrise, as Ramdas and others reported), even assuming that it had arisen as some transient. Finally, as Raschke (1957) discusses, accurate temperature measurements near ground are not easy to make. Indeed, Geiger (1965, p. 93) remarks in his classic work *The climate near the ground*, 'These results were at first accepted with some reservations, since similar conditions might be arrived at if cold air had flowed in from the environs, as from the radiative cooling of the surface of a plant' – in spite of the fact that further work by Ramdas and others (e.g. Ramanathan & Ramdas 1935) had led to the conclusion that the effect was not due to advection and not peculiar to any locality. It was perhaps the eventual confirmation by Lake (1956*a*) and Raschke (1957) that removed remaining doubts about the phenomenon. In particular Raschke made careful and extensive measurements in Poona using specially constructed radiation-compensated thermoelements (Raschke 1954), and reported very similar observations in particular from the bare and remarkably flat top of Chaturshringi Hill, where advection could not have been present. Since then it has been discovered that, contrary to the implicit suggestion of Ramdas & Atmanathan (1932), the phenomenon is not by any means confined to the tropics;

confirmatory reports have come from many parts of the world, including England (Lake 1956*a*, *b*), Canada (Oke 1970), U.S.A. (Fleagle & Badgley 1952) and in many other countries cited by Geiger (1965).

It has been suggested by a referee that the lifted temperature minimum may have been observed by Glaisher (1847). This paper includes tables that indicate that 'temperature one inch high above grass' was less than that 'on short grass' by up to a degree or two. There is no discussion by Glaisher of this point, and it is not clear why the former temperature should not be compared with that on 'long grass'. At any rate these observations are more likely to be connected with what Oke (1970) has called the 'grass-tip minimum', rather than Ramdas's observations over bare soil: the mechanisms in the two cases are different, as discussed by Oke.

There has been no completely satisfactory explanation so far of the lifted temperature minimum phenomenon observed by Ramdas. Radiation has often been thought to be responsible in some way (Lake 1956*a*, *b*; Coantic & Seguin 1971; Kondratyev 1972) but advocates for convection have not been wanting (Lettau 1979). The only quantitative analysis of the problem is due to Zdunkowski (1966), who followed up a suggestion by Möller that a haze layer above ground could lead to the strong radiative cooling that might explain the phenomenon. However, the theoretical temperature profiles presented by Zdunkowski were based on values of thermal diffusivity *lower* than the molecular value by a factor of up to 18; no results were presented for sufficiently high values of the diffusivity. In this respect these results recall a calculation made by Ramdas & Malurkar (1932) who also had to assume very low diffusivities (factor of 22) to reproduce the observed steep temperature gradients near the surface (even in the absence of a lifted minimum). Furthermore, no evidence has yet been reported of a possible haze layer when the lifted minimum occurs, e.g. in the careful and exhaustive measurements of Oke (1970), who was aware of Zdunkowski's theory. Finally, in Zdunkowski's results (e.g. in his figs 3 and 5) the minimum, when found, is rather flat, and takes the form of a nearly isothermal layer whose top coincides with the assumed upper edge of the haze layer, suggesting that the discontinuity in the modelled emissivity profile may at least in part be responsible for the prediction of the lifted minimum.

One fact from observation provides a useful clue. Raschke (1957), in his Poona experiments of 29 December 1954 over bare soil, measured wind speeds close by, and found that a lifted minimum appeared almost as soon as wind speed at 20 cm above ground dropped below 0.5 m s^{-1}, but disappeared when the speed was higher, or when a wooden lath was waved nearby (we may think of the flow as being 'tripped' by this procedure). Oke (1970) observed the lifted minimum often on bare soil, but 'harrowed soil showed only infrequent and uncertain indications of its development'. It seems clear therefore that it would not take much turbulent diffusion or soil roughness to suppress the phenomenon.

Similarly, cloud-free skies seem necessary; Ramdas & Malurkar (1932) report that with overcast conditions temperature distributions near ground are substantially different, and in particular that the gradients are much lower.

It has long been suggested that radiation, and especially its absorption and emission by water vapour (and to a lesser extent carbon dioxide: the other constituents of air are virtually transparent to the infrared wavelengths characteristic of terrestrial radiation), could play a strong role in determining temperature distributions near ground (Ramdas & Malurkar 1932; Ramanathan & Ramdas 1935; Goody 1964). If this were so, however, the mechanism involved in the formation of a lifted minimum must be subtle, as we may see by considering the two obvious limiting cases of strong and weak absorption. In the former case we may say that the photon mean free path, which is inversely proportional to the absorption (see Vincenti & Kruger 1965, p. 445), is very small. In this limit a flux-gradient relation should hold, but then radiation merely makes an additive contribution to other diffusive transport; the resulting parabolic equation cannot in general account for the lifted minimum (Vasudeva Murthy *et al.* 1991; Appendix A). If on the other hand absorption is weak, there would be no radiative cooling or heating, and once again one would be unable to explain a lifted minimum. The effect of radiation therefore demands careful analysis. In particular, we must note that the absorption coefficient of water vapour varies strongly and in a very complicated way with frequency of radiation, and the values listed, e.g. by Kondratyev (1969, p. 118), show that photon mean free paths vary from less than 10 m at wavelengths of 5.5–7 μm or 27 μm and above, to the order of 1–10 km over most of the so-called atmospheric window in the 8–14 μm band. Neither of the two limits mentioned above is therefore strictly justified.

The temperature distribution near ground may be of some importance in agricultural and horticultural applications: it affects the formation of fog and dew (Monteith 1957) and the occurrence of frost; Lake (1956*a*) quotes studies showing how tomato plants spread out on bare soil start freezing from the top. The phenomenon should also be important for retrieval of correct surface temperatures from remotely sensed radiation data.

In the rest of this paper we formulate the basic equations governing the problem adopting a suitable radiation model, and solve the resulting equations numerically after identifying the appropriate non-dimensional parameters. The nature of the solutions, and the physical mechanisms responsible for the occurrence of the lifted minimum, are then discussed. Finally the parameter values that are necessary for the occurrence of the phenomenon are delineated.

2. The energy equation

We consider calm, clear nights with no advective changes. Further, to avoid unnecessary complexity we shall assume that radiative absorption characteristics and the wind profile (needed to assess the possible effects of any residual turbulence) may be incorporated into the model as parameters; we shall discuss the surface temperature variation separately in §4. It is then a reasonable approximation to take the air temperature T as homogeneous in the horizontal plane, so that it is a function only of time t and the vertical coordinate z (figure 2). The problem is then completely governed by the one-dimensional energy equation, which may be written

$$\rho_a c_p \partial T/\partial t = -\partial Q/\partial z, \tag{2.1}$$

where ρ_a is the density of air, c_p is the specific heat at constant pressure and Q is the total energy flux, conveniently split into three components representing respectively the contributions of molecular conduction (Q_m), convection (Q_c or Q_t depending on whether free or forced) and radiation (Q_r). The molecular conduction term is simply given by

$$Q_m = -k_m \partial T/\partial z, \tag{2.2}$$

where k_m is the thermal conductivity of air. In free convection thermal transport is

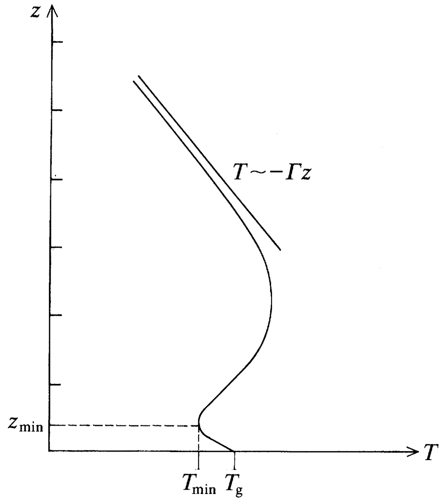


Figure 2. A schematic showing the nature of the temperature distribution under conditions of lifted minimum.

enhanced, and we shall find it sufficient for our purposes to take Q_c as a multiple of Q_m

$$Q_c = (Nu - 1)\, Q_m, \tag{2.3}$$

where Nu may be thought of as a Nusselt number, to be assumed known as a function of the appropriate Rayleigh number Ra, in the light of the extensive experimental data available (e.g. Hollands *et al.* 1975). If convection is in the forced (turbulent) regime, we shall assume that a suitable eddy conductivity k_t can be defined and replace Q_c by the term

$$Q_t = -k_t\, \partial\theta/\partial z, \quad \theta = T + \Gamma z, \tag{2.4}$$

where θ is the potential temperature, Γ being the adiabatic lapse rate ($= 9.86\ \mathrm{K\ km^{-1}}$ in dry air). Considerable work has been done on eddy diffusion in stratified flow (see Haugen 1973), suggesting that the thermal diffusivity $K_t = k_t/\rho_a c_p$ can be represented in the surface layer as

$$K_t = k_*\, U_*\, z\phi(Ri), \tag{2.5}$$

where k_* is the von Karman constant, U_* is the friction velocity and

$$Ri = \frac{k_*^2\, g z^2}{U_*^2\, \theta}\left(\frac{\partial\theta}{\partial z}\right) \tag{2.6}$$

is a Richardson number appropriate to turbulent flow in the surface layer, which is governed by the 'wall' variables U_* and z (g is the acceleration due to gravity). Among the many proposals made for the function ϕ, we find it adequate for the present purpose to adopt the relations

$$\begin{aligned} \phi(Ri) &= 1.35(1 - 9\,Ri)^{\frac{1}{2}} && \text{for } Ri < 0, \\ &= 1.35(1 + 6.35\,Ri)^{-1} && \text{for } Ri > 0; \end{aligned} \tag{2.7}$$

these are based on the data of Businger *et al.* (1971) and have been widely used (see Liou & Ou 1983). Relations (2.7) are simple to use, and also satisfy the inequality

$$\phi + Ri\, \partial\phi/\partial Ri > 0, \tag{2.8}$$

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

which ensures that the evolution equation for the temperature is governed by a parabolic equation (see Appendix A). The terms Q_c and Q_t are included chiefly to assess the role of convection in determining the intensity of the phenomenon; as we shall find that the role is not strong, a more elaborate modelling of these modes of energy transfer is not worthwhile.

The evaluation of the radiant energy flux is now considered separately.

3. The radiation model

It is a well established fact that during clear nights and in the absence of haze and other particles, the infrared cooling in the lowest 2 km of the atmosphere is chiefly due to water vapour (Liou 1980, p. 109). There are conditions (e.g. over snow) when the carbon dioxide content of air is also of some importance; this can when necessary be taken into account by a suitable enhancement of the flux emissivity of air (to be introduced below), along lines discussed by Liou (1980). The discussion below could be more generally worded in terms of a specified variation of the flux emissivity of air with height, the contributions of all constituents (water vapour, carbon dioxide, etc.) being explicitly taken into account in arriving at that specification. To avoid inessential complexity in the discussion, however, we shall find it convenient to speak of water vapour only, as it does constitute the dominant radiative absorber in general: modifications to include the effect of carbon dioxide are trivial.

Further, as we shall demonstrate below, the radiative cooling of air near the surface is strongly influenced by the emissivity of the ground. This factor has to be taken into account in any reasonable model for the energy balance near the ground.

By our assumption of horizontal homogeneity we need to consider radiative transfer only in the vertical direction. The net longwave radiative flux Q_r can then be written as

$$Q_r = F^{\uparrow} - F^{\downarrow}, \tag{3.1}$$

where $F^{\uparrow}, F^{\downarrow}$ are the upward and downward fluxes.

It is convenient to compute the radiative flux divergence in terms of the differential mass path length Δu of the absorbing gas (water vapour in the present instance) along a differential path Δz; this is given by $\Delta u = \rho_w \Delta z$, where ρ_w is the density of water vapour. Accounting for the effect of pressure variation on the absorption process in the manner suggested by Houghton (1986, p. 172), the corrected water vapour mass path length is taken as

$$u(z) = \int_0^z \rho_w(z') \left\{ \frac{p(z')}{p(0)} \right\}^{\delta} dz', \tag{3.2}$$

where $p(z)$ denotes the pressure of air at level z and δ (chosen empirically) lies in the range $0.5 < \delta < 1$. A δ of 0.9 (as recommended by Garrat & Brost (1981)) was taken for the present study.

We use the broadband flux emissivity method (Rodgers & Walshaw 1966; Liou 1980; Garrat & Brost 1981; Liou & Ou 1983), which expresses the fluxes as

$$F^{\downarrow}(u) = \int_u^{u_\infty} \sigma T^4(u', t) \frac{d\epsilon}{du'}(u' - u)\, du', \tag{3.3}$$

$$F^{\uparrow}(u) = \{\epsilon_g \sigma T_g^4(t) + (1 - \epsilon_g) F^{\downarrow}(0)\}\{1 - \epsilon(u)\} - \int_0^u \sigma T^4(u', t) \frac{d\epsilon}{du'}(u - u')\, du', \tag{3.4}$$

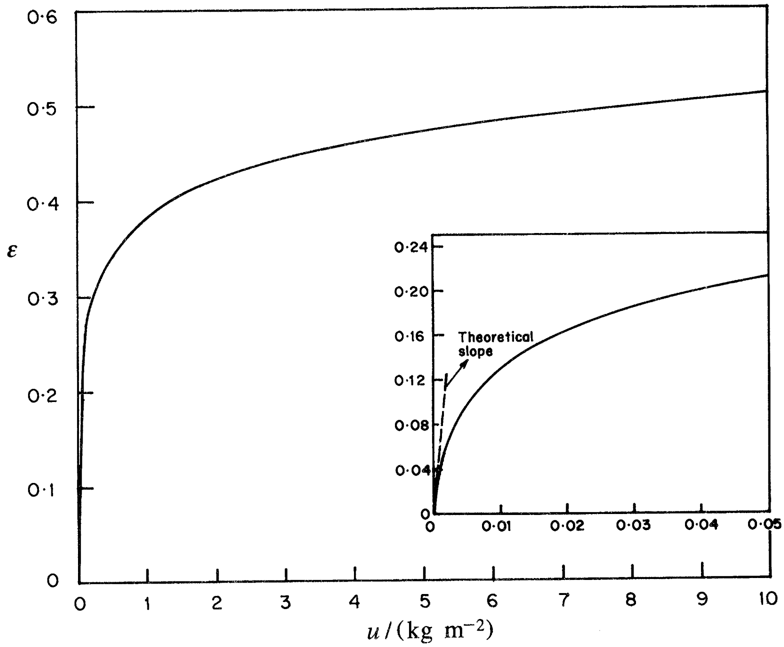


Figure 3. Variation of flux emissivity as a function of water vapour path length according to the formula of Zdunkowski & Johnson (1965), compared (in the inset) with the theoretical slope at the origin computed in Appendix B.

where $u_\infty = u(\infty)$ is the total atmospheric path-length, ϵ_g is the emissivity of ground, $T_g(t)$ is the ground temperature and $\epsilon(u)$ is the broadband flux emissivity function of water vapour, given by

$$\epsilon(u) = \frac{1}{\sigma T^4}\int_0^\infty \{1-\exp(-\kappa_w(\lambda)\,u)\}B_\lambda(T)\,d\lambda. \tag{3.5}$$

Here $\kappa_w(\lambda)$ is the spectral absorption coefficient of water vapour at wavelength λ and

$$B_\lambda(T) = (2h/\lambda^3c^2)\,[\exp(h/\lambda k_B T)-1]^{-1} \tag{3.6}$$

is the well-known Planck's function in which h is Planck's constant, c is the velocity of light and k_B is the Boltzmann constant. An essential feature of the method is that $\epsilon(u)$ is taken to be independent of T; the reason is that over the relatively narrow range of temperatures encountered within the planetary boundary layer (and *a fortiori* near the ground) the influence of T on $\epsilon(u)$ is negligible (Liou 1980, p. 111). Equation (3.4) is usually written assuming $\epsilon_g = 1$, and it is crucial for the present study that we consider the case when $\epsilon_g \neq 1$, as we shall show below.

The function $\epsilon(u)$ (shown in figure 3) is taken as

$$\begin{aligned}\epsilon(u) &= 0.04902\ln(1+1263.5u) \quad \text{for } u \leqslant 10^{-2}\ \text{kg m}^{-2},\\ &= 0.05624\ln(1+875u) \quad \text{for } u > 10^{-2}\ \text{kg m}^{-2};\end{aligned} \tag{3.7}$$

these expressions were originally proposed by Zdunkowski & Johnson (1965), and considered to apply up to $u = 0.5$ kg m^{-2}. In actual fact (3.7) is in excellent agreement with other expressions in use for larger u, such as that of Atwater (1974), used recently by Grisogno (1990): at $u = 8$ kg m^{-2}, (3.7) gives $\epsilon = 0.498$, Atwater gives 0.504. Furthermore at large u the energy balance in the atmospheric boundary layer is determined more by turbulence than by radiation, so results are not sensitive to the precise values of ϵ adopted.

Note the steep increase in ϵ near $u = 0$ displayed in figure 3; we shall find it convenient to think of the region $u < 10^{-3}$ kg m^{-2} as constituting an 'emissivity sublayer' because of this rapid variation. An inset in figure 3 shows ϵ for very small u; the derivative $\epsilon'(u)$ at $u = 0$, which plays an important role in the present theory, is 61.9 m^2 kg^{-1} according to (3.7). We have preferred (3.7) to other candidate expressions (e.g. Coantic & Seguin 1971) for the present study mainly because the above value of $\epsilon'(0)$ agrees well with the theoretical value, which is 62.9 m^2 kg^{-1} (Appendix B). Furthermore, Corradini & Severini (1975), carrying out experiments in a 50 m^3 chamber in the laboratory, found good agreement between the measured values of temperature and those calculated with the radiative contribution determined by using the expression (3.7) for $\epsilon(u)$. For all these reasons (3.7) should be entirely adequate for the present model.

4. Boundary conditions

We shall in general take the initial condition at $t = 0$ as corresponding to a constant lapse rate Γ,

$$T(z, 0) = T_{g0} - \Gamma z, \tag{4.1}$$

but some solutions with other initial conditions will also be presented.

For handling the boundary conditions at ground ($z = 0$), we pursue two different paths. In the absence of wind, the temperature at ground after sunset is determined by energy balance between radiation and heat flux from the soil:

$$k_t \frac{\partial T}{\partial z}(0, t) - F^{\uparrow} = k_s \frac{\partial T_s}{\partial z}(0, t) - F^{\downarrow}, \tag{4.2}$$

where k_s is the thermal conductivity of the soil and T_s is its temperature, governed by the conduction equation

$$\rho_s c_s \frac{\partial T_s}{\partial t} = k_s \frac{\partial^2 T_s}{\partial z^2}, \quad z < 0, \tag{4.3}$$

where ρ_s is the density of the soil while c_s is the specific heat of the soil. The initial/boundary conditions are taken to be

$$T_s(z, 0) = T_{g0}, \tag{4.4}$$

$$\frac{\partial T_s}{\partial z}(-\infty, t) = 0. \tag{4.5}$$

By solving (4.3) together with (2.1) and (4.2) one can derive the temperature distribution in air as well as soil, and the ground temperature $T_g(t)$ at $z = 0$ will also come out as a part of the solution.

We adopt the above procedure and demonstrate the solutions so obtained, but we shall find that there is an alternative but equivalent approach that is simpler and easier to interpret. This approach is based on the well-known work of Brunt (1941), who showed that to a good approximation the problem of determining ground

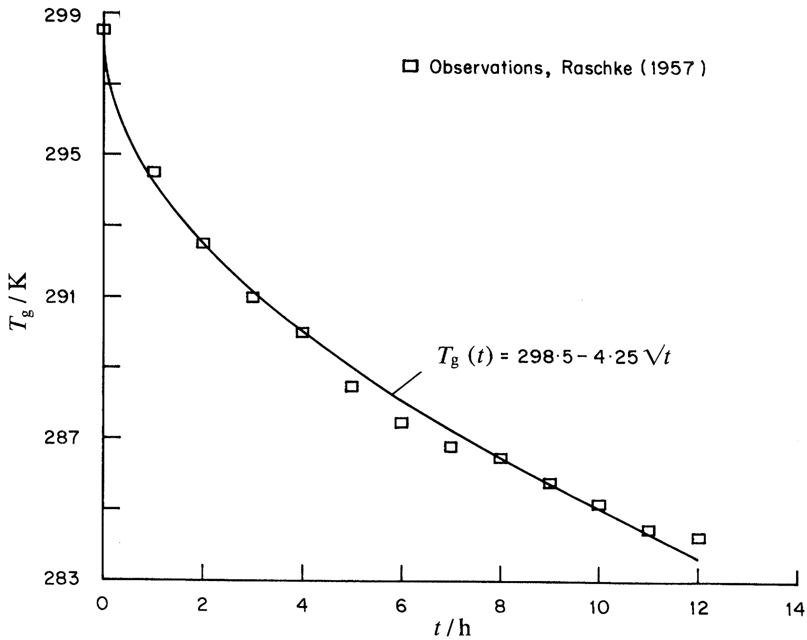


Figure 4. Comparison of observed ground temperature variation with (4.6).

temperature can be reduced to that of heat conduction in the soil subject to a constant flux boundary condition at the surface. This leads to the result that the ground temperature can be expressed as (see also Kondo 1971)

$$T_g(t) = T_{g0} - \beta\sqrt{t}, \tag{4.6}$$

where the parameter β, to be called the 'cooling rate' here for the sake of simplicity (despite the fact that its units are K $h^{-\frac{1}{2}}$), depends inversely on the square root of the thermal conductivity of the soil, k_s. This approach not only allows us to use a simple boundary condition at $z = 0$ but also enables us to parameterize soil conductivity through β. Furthermore this also avoids the need for providing a prescription of the initial temperature distribution (4.4) in the soil as no measurements of soil temperature are available under conditions of a lifted minimum. Finally equation (4.6) represents observed ground temperature distributions very well: an example is provided in figure 4, where the measurements of Raschke (1957) are seen to be well fitted by (4.6) with $T_g = 298.5$ K and $\beta = 4.25$ K $h^{-\frac{1}{2}}$. Most of the solutions we present below are obtained using the boundary condition (4.6), but in §7*c* we shall demonstrate that results so obtained are in excellent agreement with those using (4.2)–(4.5). These results have certain implications for Brunt's theory and for the important problem of prediction of ground temperature variation with time, which will be pursued separately.

Solution of the energy equation also demands a top boundary condition, which we take to be

$$\frac{\partial T}{\partial z}(\infty, t) = -\Gamma, \tag{4.7}$$

corresponding to a uniform lapse of the temperature with height (cf. figure 2). Adopting the language of matched asymptotic expansions, we may look upon this relation as the condition that matches an inner solution near the surface (over the emissivity sublayer mentioned above and discussed further in the next section) with an outer solution away from the surface characterized by a uniform lapse rate. Such an asymptotic approach to a solution of the present problem will be presented elsewhere.

5. Dimensional analysis

The dimensional parameters which are relevant to the present problem can be conveniently listed as

$$C_p = \rho_0 c_p, \sigma T_{g0}^4, T_{g0}, \beta, K_m, U_*, g, \epsilon'(0), \Gamma$$

where σ is the Stefan–Boltzmann constant and $\rho_0 = \rho_a(0)$ is the value of the air density at the surface.

We first note the existence of a radiative length scale in the problem,

$$l = [\epsilon'(0)\,\rho_0\, q_0]^{-1}, \tag{5.1}$$

where $q_0 = \rho_w(0)/\rho_0$; this is clearly a measure of the thickness of what we have called the emissivity sublayer above.

Five independent non-dimensional quantities can be formed from the above list; these are chosen here to be

$$Bo = \frac{C_p \beta \sqrt{K_m}}{\sigma T_{g0}^4}, \quad Re = \frac{U_* l}{K_m}, \quad Fr = \frac{U_*^2}{gl}, \quad \lambda = \frac{\Gamma l}{T_{g0}}, \quad \tau = \frac{\beta l}{\sqrt{K_m}\, T_{g0}}. \tag{5.2)–(5.6}$$

The parameter Bo, which is seen to be a measure of the ratio of the loss of heat in the emissivity sublayer to the radiative flux from the ground, can be thought of as a 'surface' Boltzmann number. Re is a Reynolds (more precisely Peclet) number and Fr a Froude number, both based on the emissivity sublayer thickness l and the friction velocity U_*. The parameters λ and τ are respectively a non-dimensional lapse rate and a surface cooling rate. To get an appreciation of the order of magnitude of these parameters, take $T_g = 300$ K, $q_0 = 0.01$, $U_* = O(0.01)$ m s^{-1} (see §7*c*) and $\beta = O(2 \text{ K h}^{-\frac{1}{2}})$; using the standard values (Oke 1987) $K_m = 2.5 \times 10^{-5}$ m^2 s^{-1}, $\rho_0 = 1.2$ kg m^{-3}, $c_p = 10^3$ J kg^{-1} K^{-1} and $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}, we obtain $l \approx 1.35$ m,

$$Bo \approx 10^{-4}, \quad Re \approx 5 \times 10^2, \quad Fr \approx 10^{-5}, \quad \lambda \approx 10^{-3}, \quad \text{and} \quad \tau \approx 10^{-2}.$$

The low value of the Froude number shows that gravity effects are negligible, and the modest value of the Reynolds number (for the assumed U_*) suggests that turbulence may not play a significant role: the viscous sublayer (usually taken as extending to about 10 wall units above the surface) would have a thickness of about 2.5 cm for the assumed value of U_*. The influence of the other parameters will be discussed later.

The energy equation can now be non-dimensionalized to read

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{\partial}{\partial \bar{z}}\left[(1+\bar{z} Re\, \phi(Ri)\left(\frac{\partial \bar{T}}{\partial \bar{z}} - \lambda\right)\right] - \frac{1}{Bo}\frac{\partial \bar{F}}{\partial \bar{u}}, \tag{5.7}$$

with the initial and boundary conditions discussed in §4 taking the form

$$\bar{T}(\bar{z}, 0) = \lambda \bar{z}, \quad \bar{T}(0, \bar{t}) = \sqrt{\bar{t}}, \quad \frac{\partial \bar{T}}{\partial \bar{z}}(\infty, \bar{t}) = \lambda, \tag{5.8a–c}$$

where

$$\bar{z} = z/l, \quad \bar{t} = tK_m/l^2, \quad \bar{u} = \epsilon'(0)\,u, \quad \bar{T} = (T_{g0} - T)/\tau T_{g0}.$$

Since *Bo* is small, equation (5.7) implies that the timescale of evolution of the temperature is $O(Bo)$ in the non-dimensionalization adopted, or $O(l^2/K_m Bo)$ in physical units. Given the order of magnitude of *Bo* and other non-dimensional parameters in the problem, it is natural to ask whether perturbation methods can be used; as already stated, this question will be considered separately, our present aim being to establish the physical validity of the model for describing the lifted minimum. We therefore proceed to a full numerical treatment of the problem.

6. Numerical solution

The equation derived in the last section is solved here numerically by spatial discretization in the vertical. For this purpose the upper boundary condition is specified at $L = 1$ km; this height, which at first sight may seem rather large as the phenomenon of interest occurs within 1 m above the ground, is chosen so as not to leave room for doubt about accounting accurately for radiation with long photon mean free paths. Because of the resulting wide disparity in scales we cannot afford to divide the atmosphere into horizontal layers of equal height. Instead a fine mesh is chosen near the ground and a coarse mesh near the top. The actual mesh size selected was 1 cm between 0 and 1 m, 20 cm between 1 m and 10 m, 200 cm between 10 m and 100 m and 2 m between 100 m and 1000 m. The equation is solved by the method of lines (see Graney & Richardson 1981), which results in a set of 250 coupled ordinary differential equations for the evolution of temperature in each layer. This system is solved using readily available ODE software; details may be found in Vasudeva Murthy *et al.* (1991), where it is also demonstrated that the above choice of mesh size provides a solution accurate to 0.01 K in the lowest metre of the atmosphere. The equations have to be solved to relatively high accuracy levels, as the balance between different terms in the energy equation is delicate.

When the soil conduction equation (4.3) is solved, the lower boundary condition is specified at $z = -0.5$ m. This depth is sufficient for the present purpose as it is well known (Oke 1987, p. 47) that the daily surface temperature wave is only discernible to a depth of 0.75 m for all types of soils. The mesh size in this case was taken to be 2 cm.

7. Results and discussion

(*a*) *Radiation only*

To unravel the mechanism governing the lifted minimum phenomenon, it is expedient to consider at first radiation as the only heat transfer process. We will later include conduction and turbulence to study their interaction with radiation. In these numerical experiments we shall adopt, for purposes of illustration, the values $T_{g0} = 300$ K, $\beta = 2$ K h$^{-\frac{1}{2}}$, $\epsilon_g = 0.9$ and $q_0 = 0.01$; the effect of varying these parameters will be discussed in §9.

The evolution of temperature in this case over a period of 2 h after sunset is shown in figure 5. Arrows pointing up denote the temperature of the ground while those pointing down denote the temperature of air just above the ground. We observe that in the solutions at 1 and 2 h presented in the figure, air next to ground is about 3 K cooler than ground. This discontinuity arises from the well-known radiation slip

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

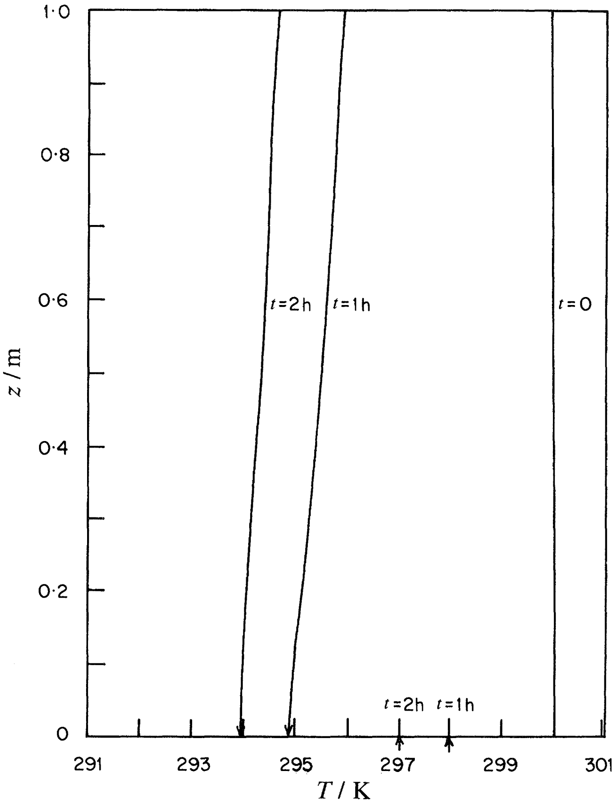


Figure 5. Evolution of temperature profiles under the influence of radiation alone ($\epsilon_g = 0.9$, $\beta = 2$ K h$^{-\frac{1}{2}}$, $q_0 = 0.01$).

effect and is basically due to long mean-free-path photons. Kondo (1971) has also shown the existence of radiative slip, but in his case the ground was at a lower temperature than air above it. For a given ground cooling rate, the magnitude of this radiative slip, defined here as $[T_g(t) - T(0+, t)]$, depends strongly on ground emissivity, as shown in figure 6.

As the radiative cooling near ground plays a key role in the lifted minimum phenomenon, it is worthwhile examining the mechanism responsible for it. To simplify matters, let us set the temperature $T(z, t) = \text{const.}$, say T_0; then the flux divergence, computed from (3.3) and (3.4), becomes

$$\partial Q_r / \partial u = \sigma T_0^4 [(1-\epsilon_g)(1-\epsilon_\infty)\epsilon'(u) + \epsilon'(u_\infty - u)]. \tag{7.1}$$

Now if ϵ_g or ϵ_∞ is identically unity, the first term in (7.1) (the contribution from the upward flux $F^{\uparrow}$) drops out. For small u the second term is approximately $\epsilon'(u_\infty)$, which is very low for large u_∞ (see figure 3), and so is also negligible. Hence the radiative flux divergence is nearly zero for air layers close to ground, which therefore experience little cooling and remain almost at the same temperature. However, following (4.6), the ground would have cooled to a lower temperature, giving a negative radiative slip. On the other hand if ϵ_g and ϵ_∞ are not too close to unity the first term in (7.1) contributes significantly near ground, because of the large magnitude of $\epsilon'(0)$; hence the air layers could cool to a temperature below that of the ground producing a positive radiative slip.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

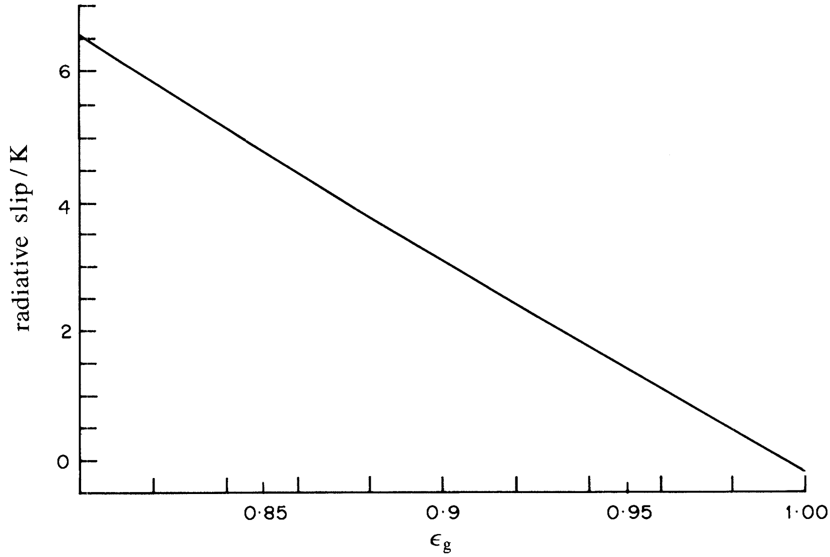


Figure 6. Variation of radiative slip with emissivity of the ground ($\beta = 2$ K h$^{-\frac{1}{2}}$, $q_0 = 0.01$).

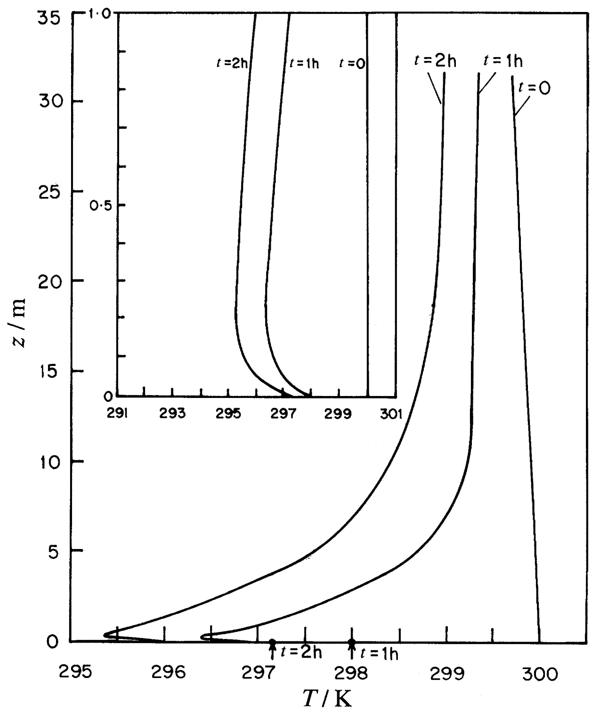


Figure 7. Computed temperature profiles under the influence of radiation and molecular conduction for heights below 35 m and 1 m (inset). Other parameters same as in figure 5.

Because of the strong role that ϵ_g plays in the present theory, it is necessary to discuss its value. It is often assumed to be unity, although compilations such as Paltridge & Platt (1976, p. 135) show that it can be rather less. Indeed Paltridge & Platt point out that the assumption that $\epsilon_g = 1$ poses problems that are not minor,

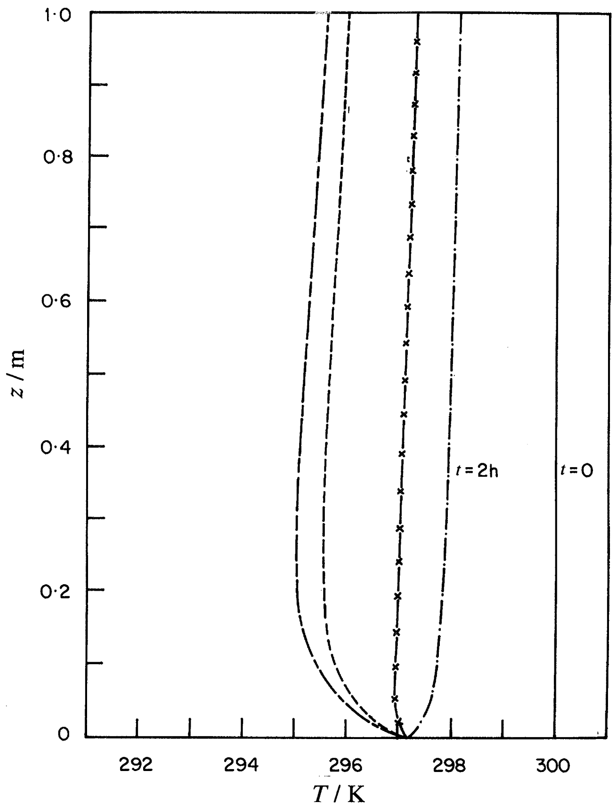


Figure 8. Computed temperature profiles in the lowest metre 2 h after sunset ($t = 0$), at different values of the friction velocity U_*, without allowing for the effect of stratification on eddy diffusivity. Other parameters same as in figure 5.

in particular because the numbers in their compilation 'refer to vertical emissivities appropriate to remote measurement of temperature by vertically oriented radiometers on spacecraft. The meterologically significant quantity is the global or flux emissivity which for natural surfaces may be much less'. Work on engineering surfaces has established that the global (also called hemispherical) emissivity is about 10% less than the normal emissivity (see, for example, Siegel & Howell 1982). Although no corresponding result is available for natural surfaces, a similar factor will presumably apply. Thus it is reasonable to assume that the global emissivities of natural surfaces will lie in the region 0.8–0.9. We shall in the following consider the range of variation of ϵ_g relevant to the present study to be 0.8–1.0, which is also the range adopted by Garrat & Brost (1981).

(b) *Effect of molecular conduction*

We consider next how the inclusion of pure molecular conduction (but no eddy transport) modifies the temperature profile with radiative slip. The evolution of temperature profiles for 2 h after sunset, with the same boundary and initial conditions and parameters as before (but with conduction included), is shown in figure 7*a*, *b*. We find that a temperature minimum appears at a height of 15 cm above the ground, and is 1.5 °C below ground temperature. Clearly conduction smears out the discontinuity observed in figure 5.

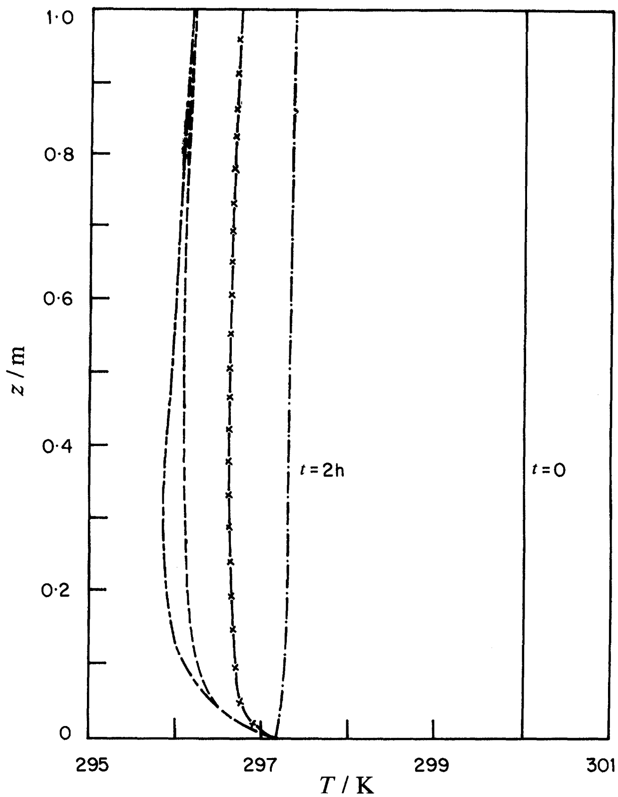


Figure 9. Computed temperature profiles in the lowest metre 2 h after sunset at different values of U_*, with allowance for stratification on eddy diffusivity. Other parameters same as in figure 5.

(*c*) *Effect of turbulence*

We consider first the effect of introduction of turbulence without allowing for stability effects, i.e. we set $\phi = 1$ in (2.5). To estimate the likely range of values of U_*, we recall the observation of Oke (1970) that during the occurrence of the lifted minimum the wind velocity at a height of 25 cm was in the range of 37–105 cm s^{-1}. This provides a very rough estimate of U_* as being in the range 2–5 cm s^{-1} if we assume that the velocity follows the classic logarithmic profile

$$U(z) = U_* \ln(z/z_0)/k_* \tag{7.2}$$

(where z_0 is the roughness height, *ca.* 0.01 cm for flat bare soil). We shall therefore take a value of $U_* = 0.1$ m s^{-1} as the upper limit of our range of interest. At $U_* \approx 10^{-3}$ m s^{-1} the viscous sublayer will have a thickness of about 25 cm, which is comparable to the usually observed height of the lifted minimum, so for U_* less than 10^{-3} m s^{-1} heat transfer is virtually dominated by molecular conduction. It is therefore seen that the relevant range of U_* is 10^{-3}–10^{-1} m s^{-1}.

Temperature profiles at $t = 2h$ for $U_* = 10^{-3}$, 10^{-2} and 10^{-1} m s^{-1} are shown in figure 8 for z less than 1 m. By comparison with figure 7 it is seen that turbulent transport warms the air, and that the structure of the lifted minimum is progressively lost as U_* increases till the temperature profiles take the form seen on most nights. For the values of ϵ_g and β chosen, the temperature minimum is on the verge of disappearance as U_* exceeds 0.01 m s^{-1}.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

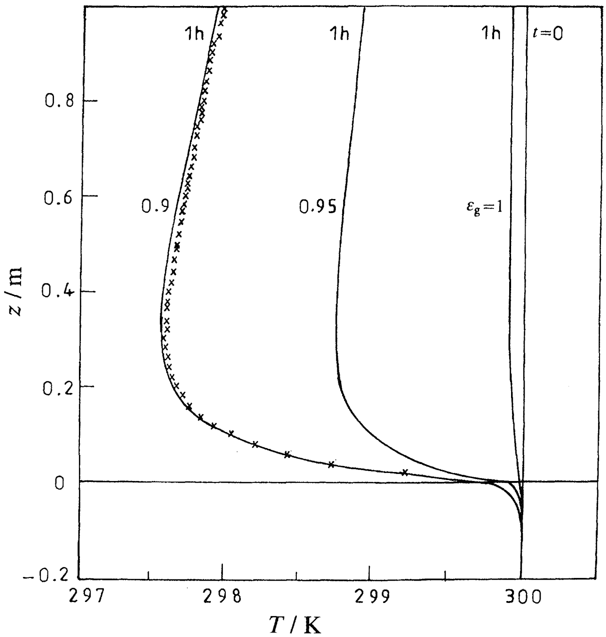


Figure 10. Computed temperature profiles obtained by solving the air–soil equations for various ϵ_g (full line); × denotes temperature profiles computed with Brunt's boundary condition at $\epsilon_g = 0.9$.

Next we include the stability function $\phi(Ri)$ and perform a similar exercise. These profiles are shown in figure 9, and are not substantially different from the previous case.

Finally in figure 10 we show temperature distributions obtained by solving the coupled soil–air problem (equations (2.1), (4.2) and (4.3) with $\rho_s = 1600$ kg m^{-3}, $c_s = 890$ J kg^{-1} K^{-1} and $k_s = 0.25$ W m^{-1} K^{-1}) for different values of ϵ_g. As ϵ_g approaches unity the minimum get weaker. This is consistent with the above results. Furthermore these temperature profiles are in excellent agreement with those computed by using Brunt's boundary condition (4.6). This is demonstrated in figure 10 for the case $\epsilon_g = 0.9$; the value of β used in this computation was obtained by fitting (4.6) to the variation of T_g predicted by the full coupled air–soil model. This clearly establishes the validity of the present approach to specifying the boundary condition at ground through the parameter β in (4.6).

8. Discussion and comparison with observations

Although there are now numerous observations of the lifted minimum, these rarely report the precise initial and boundary conditions; in particular the ground emissivity and the friction velocity at the time of measurement have never been estimated. In these circumstances, the best we can do is to make reasonable estimates for ϵ_g and U_* along lines already discussed, and offer illustrative comparisons. For this purpose we select the data of Ramanathan & Ramdas (1935), who provide good initial and boundary conditions on the temperature under wind conditions stated to have been calm (so we shall assume $U_* \approx 0$ as a first approximation). The temperature profile reported at 1800 h (local time) is taken here as the initial condition. Measured surface temperature variation soon after sunset

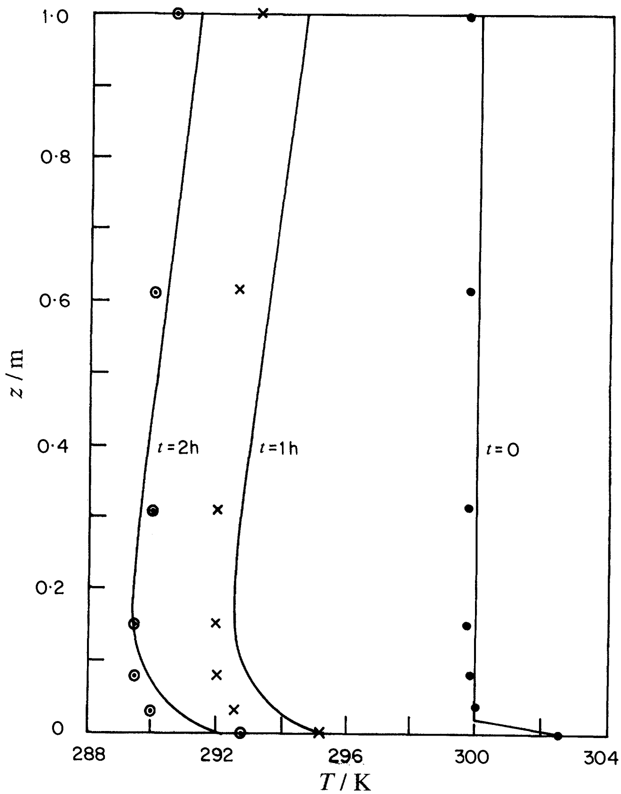


Figure 11. Comparison of temperature profiles according to present theory (full line, no eddy diffusion) with the observations (○, at $t = 0$; ×, at $t = 1$ h; ⊙, at $t = 2$ h) of Ramanathan & Ramdas (1935). Computations for $\epsilon_g = 0.8$, $\beta = 7.5$ K h^{-1} and $q_0 = 0.01$.

suggests $\beta = 7.5$ K h$^{-\frac{1}{2}}$. In figure 11 the prediction of the model with this value of β, and assuming $\epsilon_g = 0.8$ and $U_* = 0$, is compared with observations. We find that the numerical simulation is able to predict closely the temperature profile in the lowest metre of the atmosphere. Because of residual uncertainties associated with the values of the various parameters involved we should not perhaps consider the agreement shown in figure 11 as definitive, but there can be no doubt whatever that the theory predicts the right kind of behaviour.

As already pointed out, the temperature profiles predicted by the present theory (for $\epsilon_g = 0.9$, $\beta = 2$ K h$^{-\frac{1}{2}}$) do not exhibit a lifted minimum if U_* is much greater than about 0.01 m s^{-1}. At this condition the eddy diffusivity K_t at a height of 25 cm (as estimated from (2.5)) is around four times the molecular diffusivity K_m. The extreme sensitivity of the lifted minimum to the presence of turbulence that we predict here is supported by the observations of Raschke (1957). Reporting simultaneous measurements over smooth and bare soil surfaces, Oke (1970) remarks that '... the flat soil exhibits the raised minimum at all times, at a height of 2.5–25 cm above the surface, but the harrowed soil showed only infrequent and uncertain indications of its development. The raised minimum never rose above 2.5 cm and $(T_0 - T_{min})$ never exceeded $0.2C$, even with wind speeds as low as 40 cm s^{-1}'. Taking U_* at this wind speed to be about 2 cm s^{-1} and a length scale characteristic of the harrowed surface as at least 1 cm, the relevant roughness Reynolds number is about 15, which shows the surface would be aerodynamically rough (see e.g. Schlichting 1955, p. 454) and could be expected to promote at least local turbulence. It is possible that a contributing factor of the rougher surface was the slightly higher global emissivity due to radiative reflection among the roughnesses.

We still need to discuss the possibility of free convection in the layer below the lifted minimum. A Rayleigh number based on the height to the temperature minimum $z_{\min}$ and the temperature differential

$$\Delta T_{\min} = T_g(t) - T(z_{\min}, t)$$

takes a value of about 10^5 for $z_{\min} = 20$ cm, $\Delta T_{\min} = 1$ K. As this is well above the usually cited critical values of Rayleigh–Bénard instability the question arises of how the apparently unstable layer sustains itself for hours (almost till sunrise, in fact). We must, however, note that a stability analysis valid for the unusual free boundary condition at the top of the Ramdas layer is not available. A plausible explanation is that radiative transfer stabilizes this layer and raises the critical Rayleigh number Ra_c substantially. Goody (1964), Christophorides *et al.* (1970) and Vincenti & Traugott (1971) have shown that in a grey gas the critical Rayleigh number varies from its classical value for optically thin conditions to a value some three orders of magnitude higher in the optically thick limit. This extraordinary increase is easy to understand physically, for in an optically thick gas energy transport obeys a flux-gradient relationship, and a radiative diffusivity K_r can be defined; the medium therefore behaves like one with a thermal diffusivity of $K_r + K_m$, and the critical value of Ra goes up by the ratio $1 + K_r/K_m$, which can be considerable. Although some work on a quasi-grey gas has been reported by Arpaci & Gozum (1973), no detailed analysis seems to have been made of a situation more directly relevant to the present problem, which involves a non-grey gas and a semi-infinite geometry with non-monotonic temperature profiles. Nevertheless a rough estimate of the effect of radiation on Ra_c can be made using the formula (Goody 1956, p. 359; Vincenti & Traugott 1973, p. 112)

$$Ra_c \approx Ra_c(\text{diff})\{1 + t_{\text{diff}}/t_{\text{rad}}\},$$

where $Ra_c(\text{diff})$ is the classical value (i.e. when molecular diffusion alone is present) while t_{diff} and t_{rad} denote diffusion and radiation time constants. In the present problem these may be taken as

$$t_{\text{diff}} = z_{\min}^2/K_m, \quad t_{\text{rad}} = c_p\{\sigma T_{g0}^3\, \epsilon'(0)\}^{-1},$$

where t_{rad} is derived by examining the non-dimensional radiative flux divergence near the ground. Taking $z_{\min} = 20$ cm and $T_{g0} = 300$ K we obtain $t_{\text{diff}} \approx 1600$ s and $t_{\text{rad}} \approx 11$ s. If we take the cooling rate (7.1) as more relevant, the estimated t_{rad} will go up by the factor $[(1-\epsilon_g)(1-\epsilon_\infty)]^{-1}$, which is of the order 10. We thus see that Ra_c can be increased by a factor of somewhere between 10 and 150 under these conditions. While further work on stability is required to determine Ra_c more precisely under conditions corresponding to the lifted minimum, there are clearly strong reasons for believing that radiative stabilization is the chief reason for the persistence of an apparently unstable layer near the ground.

A second reason is the relatively slow increase in heat transfer rates at super-critical Rayleigh numbers. Asymptotic analysis shows that $Nu \sim Ra^{\frac{1}{3}}$ in the limit of large values of Ra. A comprehensive survey of experimental data on heat transfer between parallel plates, undertaken by Hollands *et al.* (1975), indicates that a

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

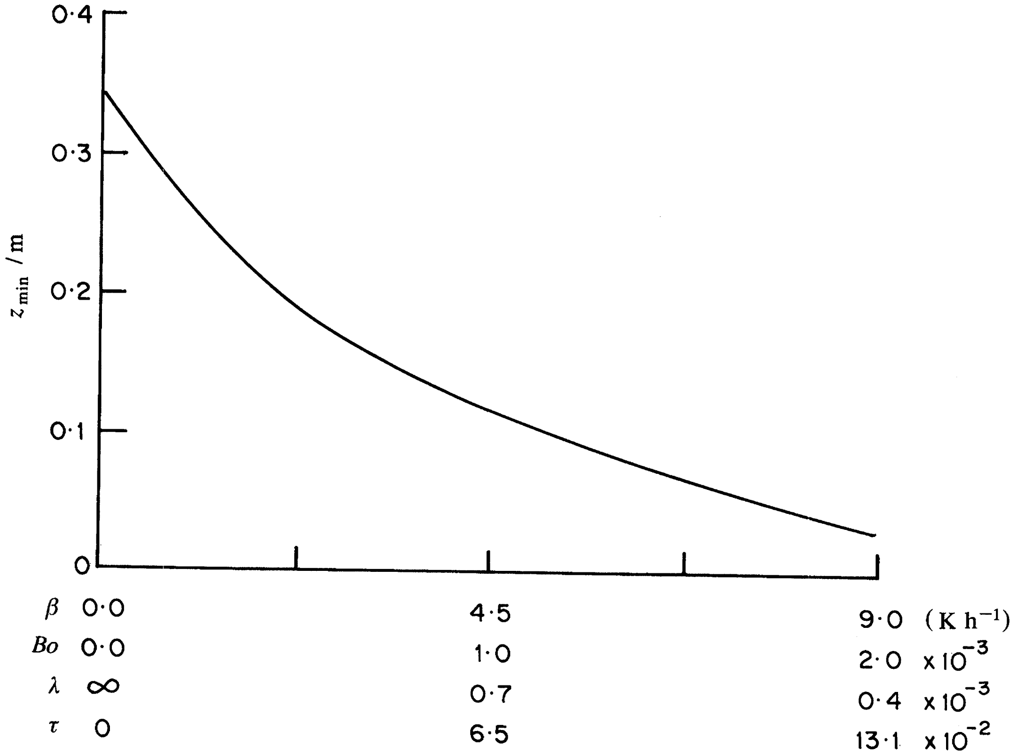


Figure 12. Variation of predicted height of temperature minimum with cooling rate ($\epsilon_g = 0.9$, $q_0 = 0.01$).

Nusselt number of 4 (which, by the results already cited, is the kind of value at which eddy transport may be expected to be sufficiently strong to destroy the lifted minimum) is attained only at a value of Ra nearly 60 times larger than critical.

We thus see that the heavier fluid at the lifted minimum can be maintained without completely overturning either solely due to radiative stabilization or in combination with the very modest increase in heat transfer even at large supercritical Rayleigh numbers.

9. Parametric study

The quantities that we shall study are the location of the minimum $z_{\min}$ and the associated 'intensity' $\Delta T_{\min}$. From the discussion in §7, it will clearly be no great loss to restrict ourselves to the radiation-conduction model, ignoring eddy transport ($Q_c = Q_t = 0$). The parameters involved are then β, Bo, λ, τ and ϵ_g. Figures 12 and 13 show the variation of $z_{\min}$ and $\Delta T_{\min}$ respectively as functions of the above parameters for $\epsilon_g = 0.9$; both quantities go to zero as β increases. We see that to observe a lifted minimum of at least 1 K above ground with emissivity 0.9 the ground cooling rate should not exceed around 4 K h$^{-\frac{1}{2}}$. Equivalently neither Bo nor λ should exceed about 10^{-3} (cf. §5). From the definition (5.2) of Bo this implies that the radiative energy flux should be at least 10^3 times a characteristic thermal diffusive flux. The low value of the parameter τ^2 shows in a different way that the surface cooling rate should be sufficiently low.

We now vary ϵ_g fixing β, Bo, λ and τ. The variation of $z_{\min}$ and $\Delta T_{\min}$ is shown in figures 14 and 15.

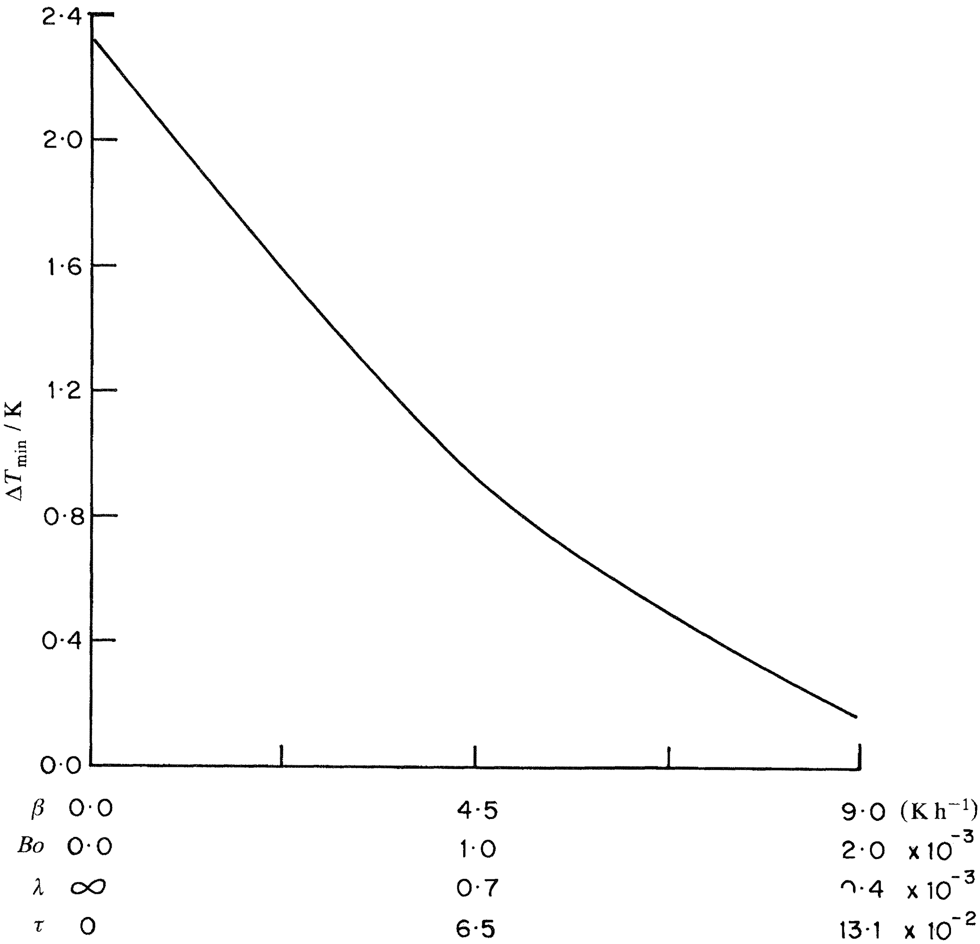
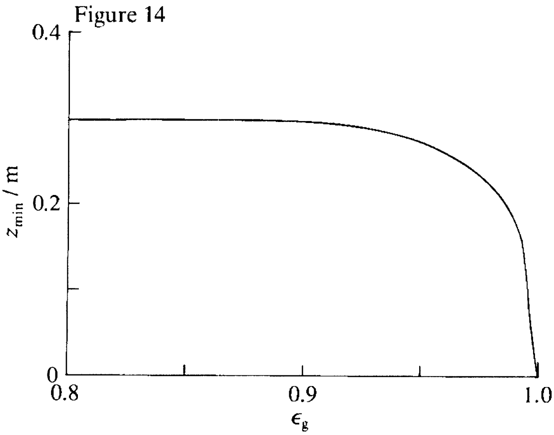


Figure 13. Variation of predicted minimum temperature with cooling rate. Other parameters same as in figure 12.

Contours of constant $z_{\min}$ and $\Delta T_{\min}$ in the (ϵ_g, β) plane are shown in figures 16 and 17. Note that as ϵ_g increases we need lower cooling rates to observe a lifted minimum. Also in these figures the region where $z_{\min}$ and $\Delta T_{\min}$ are greater than 30 cm and 3 K respectively is very small, thus demonstrating that a lifted minimum at a higher location or with greater intensity is unlikely. Most observations of the lifted minimum support this prediction (Oke (1970) quotes one instance with $z_{\min} =$ 50 cm). Curiously, the Poona measurement in the very first report of Ramdas & Atmanathan shows a deep minimum at a height of about a metre, perhaps as a result of drainage of cold air from the environs as the authors themselves suspected; but the Agra data, as well as the later measurements of Ramanathan & Ramdas (1935), are consistent with the present theory as well as other later observations.

The maps in figures 16 and 17 can be used to predict the lifted minimum phenomenon if estimates for ϵ_g and β are available.



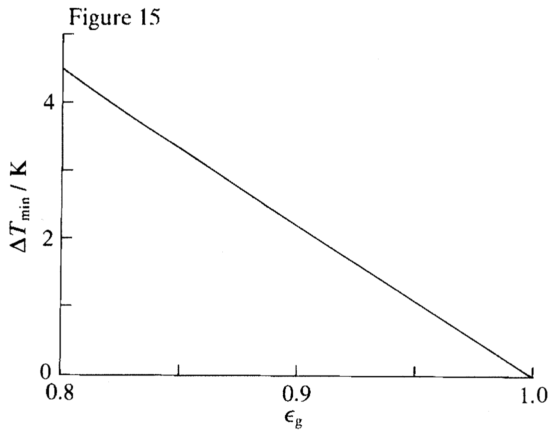
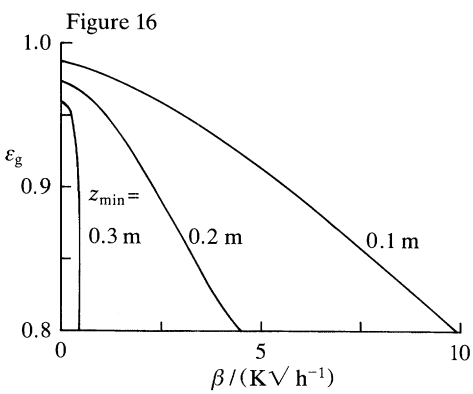


Figure 14. Variation of predicted height of temperature minimum with ground emissivity ($\beta = 2\ \mathrm{K\ h^{-\frac{1}{2}}}$, $q_0 = 0.01$).

Figure 15. Variation of predicted minimum temperature with ground emissivity. Other parameters same as in figure 14.



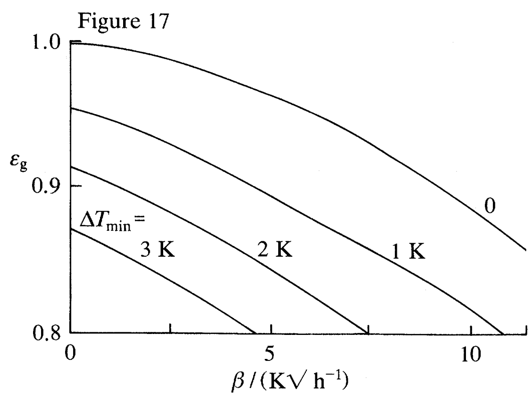


Figure 16. Contours of constant z_{min} as predicted by the present theory ($q_0 = 0.01$).

Figure 17. Contours of constant ΔT_{min} as predicted by the present theory ($q_0 = 0.01$); there is no lifted minimum above the curve labelled 0.

10. Conclusions

In the present work we have proposed a theory for the lifted minimum phenomenon in the atmosphere above bare soil on calm clear nights, without invoking phase transition in water vapour (such as fog, snow, rain or haze). The theory leads to a partial integro-differential equation that describes the energy balance between thermal diffusion and infrared radiation in the presence of water vapour, and highlights the key role played by two parameters, namely the ground emissivity and the surface cooling rate (or, equivalently, the soil conductivity). While the importance of soil conductivity has occasionally been noted (in particular by Raschke (1957)), the profound effect that even a small departure of the surface emissivity from unity can have has not been realized: the present work shows how a surface that is not perfectly black (radiatively) can substantially influence the radiative cooling near the surface. The extremely rapid variation of the absorptivity of water vapour to infrared radiation at small path lengths, in what we have called the emissivity sublayer, provides a length scale of the order of a metre that characterizes the phenomenon. Comparison with observation shows reasonable agreement, and has encouraged us to present, in the plane of the most important parameters in the problem, maps that can be used to predict when the lifted minimum occurs.

The present theory predicts that the temperature minimum is weaker if the ground is rough (which promotes turbulent diffusion and has higher emissivity) or radiatively dark (decreases aircooling), or if the soil is insulating (increases surface cooling rate). Controlled experiments in which these parameters are varied seem feasible and worthwhile in view of possible applications in agriculture and horticulture.

Further theoretical work is needed to interpret the phenomenon more directly in terms of radiative transfer in different bands of the absorption spectrum, to provide an appropriate perturbation analysis, and to assess more precisely the effect of radiation on convective instability under conditions corresponding to observed near-ground temperature distributions.

Appendix A

If we expand (2.1) using (2.2) and (2.4)–(2.7) we obtain

$$\rho_{\mathrm{a}} c_p \frac{\partial \theta}{\partial t} = \left[K_{\mathrm{m}} + k_* U_* z \left\{\phi + Ri \frac{\partial \phi}{\partial Ri}\right\}\right] \frac{\partial^2 \theta}{\partial z^2} + \dots .$$

This equation is parabolic if the quantity in $[\cdot]$ is positive. On the other hand if it is negative it would be equivalent to solving the heat conduction equation backwards in time, which is well known to be ill-posed. Clearly if inequality (2.8) holds then the above equation is parabolic and consequently well-posed.

Appendix B

The quantity $\epsilon'(0)$ plays an important role in the present analysis of the lifted minimum phenomenon (see (5.1)). The $\epsilon(u)$ used in the present work (equation (3.7)) is based on laboratory measurements of infrared radiation transmission through a given path length u. Unfortunately it is difficult to measure transmission for path lengths below 10^{-3} kg m^{-2}; $\epsilon(u)$ has therefore generally been prescribed empirically in this region, leading to widely different values of $\epsilon'(0)$ among the available proposals. Thus Coantic & Seguin's (1971) expression for $\epsilon(u)$ gives $\epsilon'(0) \approx 27$ m^2 kg^{-1}, whereas (3.7) gives $\epsilon'(0) = 61.9$ m^2 kg^{-1}. In this appendix we shall calculate $\epsilon'(0)$ from the available spectral data.

From (3.5) we obtain

$$\epsilon'(0) = \frac{1}{\sigma T^4} \int_0^\infty \kappa_{\mathrm{w}}(\lambda) B_\lambda(T) \, \mathrm{d}\lambda,$$

which can be approximated by the sum

$$\epsilon'(0) = \sum_i \frac{1}{\sigma T^4} \int_{\lambda_i}^{\lambda_i + \Delta\lambda_i} \kappa_{\mathrm{w}}(\lambda) B_\lambda(T) \, \mathrm{d}\lambda, \qquad \text{(A 1)}$$

where $\Delta\lambda_i$ is the width of the spectral interval $(\lambda_i, \lambda_i + \Delta\lambda_i)$ such that κ_{w} can be considered to be a constant ($=\kappa_{wi}$) within the interval. Using this approximation we obtain

$$\epsilon'(0) \approx \sum_i \kappa_{wi} f_i(T), \tag{A 2}$$

where

$$f_i(T) = f_b(\lambda_i + \Delta\lambda_i, T) - f_b(\lambda_i, T),$$

$$f_b(\lambda_i, T) \equiv \frac{1}{\sigma T^4} \int_0^{\lambda_i} B_\lambda(T)\,d\lambda.$$

The values of κ_{wi} for the various water vapour bands are given in Liou (1980). The spectral data for the rotation and the vibration–rotation bands are based on the statistical band model of Goody, while for the 8–12 μm band they are based on the data of Roberts, Selby & Biberman (Liou 1980, p. 108). The values of $f_b(\lambda_i, T)$ are well tabulated in Siegel & Howell (1972, p. 739). Using these data in (A 2) we obtain $\epsilon'(0) \approx 62.9\ m^2\ kg^{-1}$.

The authors are grateful to Professors J. L. Monteith and J. V. Lake for illuminating discussions. A.S.V.M. thanks Dr S. Kesavan for his valuable advice.

References

Arpaci, V. S. & Gozum, D. 1973 Thermal stability of radiating fluids: the Bénard problem. *Phys. Fluids* **16**, 581–588.

Brunt, D. 1941 *Physical and dynamical meteorology*, 2nd edn. Cambridge University Press.

Businger, J. A., Wyngaard, J. C., Izumi, Y. & Bradley, E. F. 1971 Flux profile relationship in the atmospheric surface layer. *J. atmos. Sci.* **28**, 181–189.

Christophorides, C. & Davis, S. H. 1970 Thermal instability with radiative transfer. *Phys. Fluids* **13**, 222–226.

Coantic, M. 1978 An introduction to turbulence in geophysics, and air sea interactions. AGARD Report AG 232, NATO.

Coantic, M. & Seguin, B. 1971 On the interaction of turbulent and radiative transfer in the surface layer. *Boundary Layer Meteorology* **3**, 152–177.

Corradini, C. & Severini, M. 1975 Laboratory experimental check of radiative air cooling theory. *Q. Jl R. met. Soc.* **101**, 163–167.

Fleagle, R. G. & Badgley, F. I. 1952 The nocturnal cold layer. Occasional Rep. no. 2, University of Washington, Atmospheric Turbulence study AT 45-1.

Garrat, J. R. & Brost, R. A. 1981 Radiative cooling effects within and above the nocturnal boundary layer. *J. atmos. Sci.* **38**, 2730–2746.

Geiger, R. 1965 *The climate near the ground*. Harvard University Press.

Glaisher, J. 1847 On the amount of the radiation of heat, at night, from the Earth, and from various bodies placed on or near the surface of the Earth. *Phil. Trans. R. Soc. Lond.* **37**, 119–216.

Goody, R. M. 1964 *Atmospheric radiation*. Part I. *Theoretical basis*. Oxford: Clarendon Press.

Graney, L. & Richardson, A. A. 1981 The numerical solution of non-linear partial differential equations by the method of lines. *J. comput. appl. Math.* **7**, 229–236.

Grisogno, B. 1990 A mathematical note on the slow diffusive character of the long-wave radiative transfer in the stable atmospheric nocturnal boundary layer. *Boundary Layer Meteorology* **52**, 221–225.

Haugen, D. A. 1973 *Workshop on micrometeorology*. Boston, MA: American Meteorological Society.

Hollands, K. G. T., Raithby, G. D. & Konicek, L. 1975 Correlation equations for free convection heat transfer in horizontal layers of air and water. *Int. J. Heat Mass Transfer* **18**, 879–884.

Houghton, J. T. 1986 *The physics of atmospheres*. Cambridge University Press.

Kondratyev, K. Ya. 1965 *Radiative heat exchange in the atmosphere*. New York: Pergamon Press.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Kondraytev, K. Ya. 1972 Radiation processes in the atmosphere. WMO Note 309.

Kondo, J. 1971 Effect of radiative heat transfer on profiles of wind, temperature and water vapour in the atmospheric boundary layer. *J. met. Soc. Japan* **9**, 75–94.

Lake, J. V. 1956*a* The temperature profile above bare soil on clear nights. *Q. Jl R. met. Soc.* **82**, 187–197.

Lake, J. V. 1956*b* Discussion on the paper of Lake. *Q. Jl R. met. Soc.* **82**, 530–531.

Lettau, H. H. 1979 Wind and temperature profile prediction for diabatic surface layers including strong inversion cases. *Boundary Later Meteorology* **17**, 443–464.

Liou, K.-N. 1980 *An introduction to atmospheric radiation*. New York: Academic Press.

Liou, K.-N. & Ou, S.-C. 1983 Theory of equilibrium temperatures in radiative-turbulent atmospheres. *J. atmos. Sci.* **40**, 214–229.

Monteith, J. L. 1957 Dew. *Q. Jl R. met. Soc.* **83**, 322–341.

Oke, T. R. 1970 The temperature profile near the ground on calm clear nights. *Q. Jl R. met. Soc.* **96**, 14–23.

Oke, T. R. 1987 *Boundary layer climates*. London: Methuen.

Paltridge, G. W. & Platt, C. M. R. 1976 *Radiative processes in meteorology and climatology*. Amsterdam: Elsevier Scientific Publishing Company.

Ramanathan, K. R. & Ramdas, L. A. 1935 Derivation of Angstrom's formula for atmospheric radiation and some general considerations regarding nocturnal cooling of air layers near the ground. *Proc. Ind. Acad. Sci.* **1**, 822–829.

Ramdas, L. A. & Atmanathan, S. 1932 The vertical distribution of air temperature near the ground at night. *Beit. Geophys.* **37**, 116–117.

Ramdas, L. A. & Malurkar, S. L. 1932 Theory of extremely high lapse rates of temperature very near the ground. *Ind. J. Phys.* **6**, 495–508.

Raschke, K. 1954 A sturdy thermoelectric psychrometer for microclimatic measurements. *Proc. Ind. Acad. Sci.* A **39**, 98–107.

Raschke, K. 1957 Über das nächtliche Temperaturminium über nackten Boden in Poona. *Met. Rundschau.* **10**, 1–11.

Rodgers, C. D. & Walshaw, C. D. 1966 The computation of infrared cooling rate in planetary atmospheres. *Q. Jl R. met. Soc.* **92**, 67–92.

Schlichting, H. 1955 *Boundary layer theory*. London: Pergamon.

Siegel, R. & Howell, R. J. 1982 *Thermal radiation heat transfer*. New York: Hemisphere.

Sutton, O. G. 1953 *Micrometeorology*. New York: McGraw-Hill.

Vasudeva Murthy, A. S., Srinivasan, J. & Narasimha, R. 1991 Modelling the lifted minimum phenomenon in the atmosphere. Report no. 91 AS 1, Centre of Atmospheric Sciences, Indian Institute of Science, Bangalore, India.

Vincenti, W. G. & Kruger, C. H. 1965 *Introduction to physical gas dynamics*. New York: John Wiley.

Vincenti, W. G. & Traugott, S. C. 1971 The coupling of radiative transfer and gas motion. *A. Rev. Fluid Mech.* **3**, 89–116.

Zdunkowski, W. 1966 The nocturnal temperature minimum above the ground. *Beitr. Phys. Atmos.* **39**, 247–253.

Zdunkowski, W, & Johnson, F. G. 1965 Infrared flux divergence calculations with newly constructed radiation tables. *J. appl. Met.* **4**, 371–377.

Received 11 September 1991; revised 4 September 1992; accepted 9 November 1992

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES